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On the equivalence between real and superfield 5D formalisms

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ABSTRACT: We explicitly construct a dictionary between two different supersymmetric formalisms for five-dimensional theories commonly used in the literature within a particular model where the bulk piece is off-shell while the brane action is on-shell. One of these models is the real formalism, which consists in doubling the number of degrees of freedom and then imposing reality constraints and the other is the usual superfield formalism.

KEYWORDS: Supersymmetry Breaking, Extended Supersymmetry

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1 Introduction

The supersymmetry is a graded extension of the Poincaré spacetime symmetry relating bosons and fermions in a single multiplet [1, 2]. A powerful supersymmetry representation was introduced by Salam and Strathdee in 1974 [3] and it is known as the superspace formulation, quickly developed by many authors [4]. It consists of a representation of the supersymmetric algebra on the so called superspace: a manifold spanned by the usual spacetime coordinates plus a set of anticommuting spinorial variables $(\theta, \bar{\theta})$. The irreducible representations of supersymmetry are now functions on the superspace called superfields [5], which contain the dynamical degrees of freedom plus auxiliary fields furnishing a representation of the superalgebra, and invariant actions can be easily constructed projecting the (product of) superfields onto $\theta^2\bar{\theta}^2$ component or θ^2 ($\bar{\theta}^2$) if they are chiral (anti-chiral) superfields [6]. For extended supersymmetries, $N \geq 2$, the irreducible representations can be split into $N = 1$ superfields and invariant actions can be constructed the same way by requiring them to respect the global R -symmetry of the algebra. For instance, in 5D $N = 1$ supersymmetry, which corresponds to $N = 2$ from the 4D point of view, the matter hypermultiplet can be decomposed into two 4D-chiral superfields¹ while

¹This is not always the case. For $N = 1$ in 6D there is no off-shell formulation for the (massive) matter hypermultiplet. The reason is that the minimal (massive) multiplet with 1/2 as maximum helicity in 6D should be equivalent to an 5D $N = 2$ multiplet, and hence, charged under a central generator of the algebra. However, there is no central charge for $N = 1$ in 6D [7]. For massless representations, however, the central charge realizes trivially on the physical states.

the vector hypermultiplet can be expressed as one 4D-chiral superfield and one 4D-vector superfield [8].

The aim of the present paper is to translate to the superfield language a particular bulk-boundary model [9] for ElectroWeak symmetry breaking, where an off-shell-on-shell real formulation is used. In the real formalism [10, 11] the (irreducible) supermultiplet contains twice the number of fields but the latter are subject to reality constraints.² More precisely, the model considered in ref. [9] is a five dimensional ($N = 1$) supersymmetric matter action defined on a 5D manifold with 4D boundaries, where the bulk piece is off-shell while the boundary part is on-shell. The breaking of supersymmetry comes from the boundary term and the action for the lowest modes turns out to be MSSM-like. Furthermore, it is shown that for certain bulk-boundary configurations the scalar spectrum allows a tachyonic lowest mass at the tree level, which has been later used for triggering the ElectroWeak symmetry breaking (with radiative corrections) within the same model [13], although developed in the superfield formulation, where invariant interaction terms are easily incorporated. Nevertheless, and albeit the supersymmetry is assured for certain bulk-boundary arrangement, the on-shell-off-shell formalism does not allow a completely trivial superfield reformulation, as we shall see. The current work, thus, consists in giving a dictionary connecting the particular formulations used in refs. [9, 13], which helps to better understand the nature of the proposed supersymmetry breaking mechanism by boundary terms and clarifies certain technical aspects of the model.

The paper is organized as follows: In section 2 we will quickly review the model proposed in ref. [9]. In section 3 the real-to-superfield dictionary will be worked out, that is: the invariant action will be rewritten in terms of superfields and section 4 will be devoted to the supersymmetry breaking pattern. Finally, in section 5 we present our conclusions and an appendix is added at the end of the paper just containing some more technical details of the translation.

2 Real formalism revisited

As it was mentioned in the introduction, the model is defined in a flat 5D manifold with boundaries: $\Sigma = M_4 \times I$, M_4 being the 4D Minkowski space, I the interval $[0, \pi R]$ and R the compactification radius. The metric signature is taken as $\eta_{MN} = \text{diagonal}(+1, -1, -1, -1, -1)$ and the field content of the hypermultiplet in 5D is (Φ_i, Ψ, F_i) where Φ_i are complex scalars and F_i auxiliary fields, both transforming as doublets of $SU(2)_R$ while Ψ is a Dirac fermion. To have a manifest $SU(2)_R$ covariance in the superalgebra we use the $N = 2$ 5D structure [11]

$$\{Q_i, Q_j\} = \epsilon_{ij} \gamma^M C P_M + \epsilon_{ij} Z C, \tag{2.1}$$

subject to a symplectic Majorana (SyM) constraint

$$\bar{Q}^i \equiv Q_i^\dagger \gamma^0 = \epsilon^{ij} Q_j^T C, \tag{2.2}$$

²In refs. [12] a general correspondence for the off-shell formulation of supergravity, can be found.

where ϵ^{ij} is the total antisymmetric tensor and

$$C = -\mathbf{1} \otimes i\sigma_2 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \quad (2.3)$$

is the 5D charge conjugation matrix verifying $C\gamma^M C = -(\gamma^M)^T$. In addition $\gamma^M = (\gamma^\mu, \gamma^5)$ with³ $\gamma^{\dot{5}} = -i\gamma^5$ and $\gamma^5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$. Finally, P_M are the spacetime translation generators, Z is a central charge and consistency with (2.2) imposes Z to be hermitian. Now it is clear how the real formalism is implemented: we double the number of degrees of freedom and impose reality constraints, that is

$$\mathbb{H}^\alpha = (\Phi_i, \Psi, F_i)^\alpha, \quad (2.4)$$

where i is an $SU(2)_R$ index while α is an extra $SU(2)_H$ index. Those constraints read

$$\bar{\Psi}_\alpha \equiv (\Psi^\alpha)^\dagger \gamma^0 = \epsilon_{\alpha\beta} (\Psi^\beta)^T C, \quad (2.5)$$

$$\bar{\Phi}_\alpha^i \equiv (\Phi_i^\alpha)^* = \epsilon^{ij} \epsilon_{\alpha\beta} \Phi_j^\beta, \quad (2.6)$$

which can be compactly written as

$$\begin{aligned} \bar{\Psi} &= -\Psi^T \epsilon \otimes C, \\ \bar{\Phi}^* &= \epsilon \otimes \epsilon \Phi. \end{aligned}$$

The auxiliary fields verifying the same constraint as the scalars. In (2.6) $\epsilon_{\alpha\beta}$ is again the total antisymmetric tensor and in both cases (H and R) the convention taken is such that $\epsilon^{12} = \epsilon_{12} = 1$. The action is given by

$$\begin{aligned} \mathcal{S} &= \int_\Sigma \left(-\frac{1}{2} \bar{\Phi} \partial^2 \Phi + \frac{i}{2} \bar{\Psi} \gamma^M \partial_M \Psi + 2\bar{F}F + 2i\bar{F}\mathcal{M}\Phi + \frac{1}{2} \bar{\Psi}\mathcal{M}\Psi \right) \\ &+ \int_{\partial\Sigma} \left(\frac{1}{4} \bar{\Psi} S_f \Psi + \frac{1}{4} (\bar{\Phi} R_f \Phi)' + \frac{1}{4} \bar{\Phi} N_f (-\mathbf{1} + R_f) \Phi \right), \end{aligned} \quad (2.7)$$

where $\mathcal{M}, S_f, R_f \equiv T_f \otimes S_f$ are hermitian matrices and N_f are real constants, \mathcal{M} and S_f act on $SU(2)_H$ indices while T_f act on $SU(2)_R$. The subscript f takes the values $0, \pi$ and indicates the boundary and the prime stands for the derivative with respect to the fifth coordinate. The reality constraints ensure the reality of the kinetic term since

$$\begin{aligned} \bar{\Psi} \gamma^M \partial_M \Psi &= -\Psi^T C \gamma^M \partial_M \epsilon \Psi = \partial_M \Psi^T \epsilon \otimes C \gamma^M \Psi = -\partial_M \bar{\Psi} \gamma^M \Psi, \\ \bar{\Phi} \partial^2 \Phi &= \partial^2 \bar{\Phi} \Phi, \end{aligned} \quad (2.8)$$

and for the rest of the terms to be real it is required that $\mathcal{M}, S_f, T_f \in su(2)$, thus, we can define (dimensionless) 3-vectors $\vec{p}, \vec{s}_f, \vec{t}_f$ such that $\mathcal{M} = M \vec{p} \cdot \vec{\sigma}$ and $S_f = \vec{s}_f \cdot \vec{\sigma}, T_f = \vec{t}_f \cdot \vec{\sigma}$, where M is a constant⁴ with dimension of energy and $\vec{\sigma}$ are the Pauli matrices.

³The dotted index stands for a Lorentz one and hence $\gamma^{\dot{5}} = -\gamma_5$

⁴ M can be suitably redefined such that \vec{p} is a unit vector.

2.1 Equations of motion and boundary conditions

The variational principle applied to the bulk + brane action yields a 5D variation (whose vanishing yields the equations of motion) plus a boundary one, the latter being

$$\frac{1}{2} \int_{\partial\Sigma} \delta\bar{\Psi} \left(-i\gamma^{\dot{5}} + S_f \right) \Psi + \delta\bar{\Phi}' (\mathbf{1} + R_f) \Phi + \delta\bar{\Phi} (-\mathbf{1} + R_f) [\Phi' + N_f \Phi] ,$$

and thus yielding the boundary conditions

$$\left(\mathbf{1} - i\gamma^{\dot{5}} \otimes S_f \right) \Psi \Big|_f = 0, \tag{2.10}$$

$$(\mathbf{1} + R_f) \Phi \Big|_f = 0, \tag{2.11}$$

$$(\mathbf{1} - R_f) [\Phi' + N_f \Phi] \Big|_f = 0. \tag{2.12}$$

A necessary condition for those restrictions to allow a non trivial solution is that the matrices

$$\mathbf{1} + i\gamma^{\dot{5}} \otimes S_f$$

and

$$\begin{pmatrix} \mathbf{1} + R_f & 0 \\ N_f (\mathbf{1} - R_f) & \mathbf{1} - R_f \end{pmatrix},$$

must be singular.⁵ Their determinants are easily found to be $(1 - |\vec{s}_f|^2)^2$ and $(1 - |\vec{s}_f| |\vec{t}_f|)^4$, respectively, and hence $|\vec{s}_f| = |\vec{t}_f| = 1$. On the other hand, the equations of motion for the auxiliary fields are

$$F = -\frac{i}{2} \mathcal{M} \Phi, \quad \bar{F} = \frac{i}{2} \bar{\Phi} \mathcal{M}. \tag{2.13}$$

2.2 Supersymmetry of the action and boundary conditions

The realization of the supersymmetric algebra (2.1) at the level of the fields reads

$$\begin{aligned} \delta_\chi \Phi_i^\alpha &= i\bar{\chi}_i \Psi^\alpha, \\ \delta_\chi \Psi_\alpha &= -\gamma^M \chi^i \partial_M \Phi_i^\alpha + 2\chi^i F_i^\alpha, \\ \delta_\chi F_i^\alpha &= -\frac{i}{2} \bar{\chi}_i \gamma^M \partial_M \Psi^\alpha, \end{aligned} \tag{2.14}$$

whose parameter satisfies an analogous symplectic Majorana reality constraint⁶

$$\bar{\chi}^i \equiv (\chi_i)^\dagger \gamma_0 = \epsilon^{ij} \chi_j^T C. \tag{2.15}$$

As we show in appendix A the boundary conditions are stable under supersymmetry if, and only if, $(i\gamma^{\dot{5}} - T_f^T) \chi = 0$ and $N_f = M \vec{p} \cdot \vec{s}_f$, where the projection on the supersymmetric parameters forces $\vec{t}_0 = \vec{t}_\pi$ in order to have a non vanishing residual supersymmetry.

⁵An exhaustive study of the spectrum allowed by these boundary conditions is made in ref. [9].

⁶Indeed it is needed to ensure the consistency of the supersymmetric algebra with the reality constraints of the fields.

Concerning the supersymmetry of the action, the bulk and boundary pieces are not separately invariant, instead the bulk action varies into a total derivative which after partial integration combines with the boundary variation to give

$$\int_{\partial\Sigma} \left[2i\bar{\Psi}\gamma^{\dot{5}}\chi \left(F + \frac{i}{2}\mathcal{M}\Phi \right) + \frac{1}{2}\bar{\Phi}(\mathbf{1} + R)\delta_{\chi}\Phi' + \frac{1}{2}(\bar{\Phi}' + N\bar{\Phi})(-\mathbf{1} + R)\delta_{\chi}\Phi + \frac{1}{2}\bar{\Psi}(-i\gamma^{\dot{5}} + S)\delta_{\chi}\Psi \right], \quad (2.16)$$

which cancels upon the use of boundary conditions, provided they are stable under supersymmetry, and the equations of motion for the auxiliary fields. This is to be expected since being the boundary term on-shell,⁷ the supersymmetry requires the boundary conditions to be satisfied [14], this will be explicitly shown within our case in the next section.

On the other hand, the breaking of supersymmetry takes place on the boundaries, whose role is to determine the subspace of possible configurations the fields can lie on, and thus it is a spontaneous breaking, which will be checked in section 4.

3 Superfield description

The real formalism is suitable to make contact between an interval approach with boundary mass matrices and an orbifold model with odd bulk masses. However it is convenient to translate this formalism into superfield language where the coupling terms are easily implemented.

Now to recast the action in superfields we will first consider the case $T_0 = T_{\pi} \equiv T$, while $N_f = \vec{p} \cdot \vec{s}_f M$, according to what we saw previously and for simplicity we will take⁸ $T = -\sigma_3$ and $\vec{p}_0 = (0, 0, 1)$, the reason for that choice of signs will become clear in a moment.

Furthermore, the reality constraints (2.5), (2.6) and (2.15) can be solved as

$$\begin{aligned} \Phi &= \begin{pmatrix} \Phi_1^1 \\ \Phi_2^1 \\ -\Phi_2^{1*} \\ \Phi_1^{1*} \end{pmatrix}, \\ \Psi^1 &= \begin{pmatrix} \psi_L^1 \\ \bar{\psi}_R^1 \end{pmatrix}, \\ \Psi^2 &= \begin{pmatrix} -\psi_R^1 \\ \bar{\psi}_L^1 \end{pmatrix}, \end{aligned} \quad (3.1)$$

⁷There is no auxiliary field present although it is a mass term.

⁸Notice that we can always do so by means of global rotations of $SU(2)_R$ and $SU(2)_H$, respectively, although T can not be connected with $-T$ by any unitary transformation.

$$\begin{aligned}\chi_1 &= \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix}, \\ \chi_2 &= \begin{pmatrix} -\eta \\ \bar{\xi} \end{pmatrix},\end{aligned}\tag{3.2}$$

and, as it is shown in appendix B, the fields can be split into two chiral multiplets according to

$$W_c = \phi_c + \sqrt{2}\theta\psi_c + F_c\theta^2,\tag{3.3}$$

$$W = \phi + \sqrt{2}\theta\psi + F\theta^2,\tag{3.4}$$

upon the redefinitions

$$\begin{aligned}\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} &\equiv \begin{pmatrix} -i\Phi_1^1 \\ -i\Phi_1^2 \end{pmatrix} \equiv \varphi, \\ \begin{pmatrix} F_c \\ F \end{pmatrix} &\equiv \begin{pmatrix} -2F_2^1 + \partial_5\phi^* \\ -2F_1^{1*} - \partial_5\phi_c^* \end{pmatrix} \equiv \mathcal{F}, \\ \begin{pmatrix} \psi_c \\ \psi \end{pmatrix} &\equiv \begin{pmatrix} \psi_L^1 \\ -\psi_R^1 \end{pmatrix}.\end{aligned}$$

Accordingly, the equations of motion for the auxiliary fields can be compactly expressed as

$$\mathcal{F} = \epsilon [\varphi'^* + M\sigma_3\varphi^*] = \epsilon [\varphi' + M\vec{p}_0 \cdot \vec{\sigma} \varphi]^*,\tag{3.5}$$

with ϵ the total antisymmetric 2-tensor, while the fermionic boundary conditions (2.10) translate into

$$(\mathbf{1} - S) \begin{pmatrix} \psi_c \\ \psi \end{pmatrix} = 0.\tag{3.6}$$

The bosonic sector takes the form

$$(\mathbf{1} - \sigma_3 \otimes S) \begin{pmatrix} \varphi \\ -\epsilon\varphi^* \end{pmatrix} = 0,\tag{3.7}$$

$$(\mathbf{1} + \sigma_3 \otimes S) \left[\begin{pmatrix} \varphi' \\ -\epsilon\varphi'^* \end{pmatrix} + M\vec{s} \cdot \vec{p}_0 \begin{pmatrix} \varphi \\ -\epsilon\varphi^* \end{pmatrix} \right] = 0,\tag{3.8}$$

or equivalently⁹

$$(\mathbf{1} - S) \varphi = 0,\tag{3.9}$$

$$(\mathbf{1} + S) [\varphi' + M\vec{s} \cdot \vec{p}_0 \varphi] = 0.\tag{3.10}$$

Finally, using the identity

$$M \vec{s} \cdot \vec{p}_0 \mathbf{1} = \frac{1}{2} \{S, \mathcal{M}_0\},$$

⁹Now it is clear why we chose $T = -\sigma_3$ although such election is not arbitrary since it affects the projector of the supersymmetry parameters (see appendix B).

with $\mathcal{M}_0 = M\vec{p}_0 \cdot \vec{\sigma}$, and that

$$\varphi = \frac{1}{2}(\mathbf{1} + S)\varphi,$$

eq. (3.10) becomes

$$(\mathbf{1} + S)\varphi' + (S\mathcal{M}_0 + \mathcal{M}_0S)\varphi = 0.$$

Now adding and subtracting $\mathcal{M}_0\varphi$ and using eq. (3.9) we are left with

$$(\mathbf{1} + S)[\varphi' + \mathcal{M}_0\varphi] = 0. \quad (3.11)$$

Taking then the complex conjugate of (3.11) and using the identities $\epsilon S \epsilon = S^*$ and $\epsilon^2 = -\mathbf{1}$ we finally find

$$0 = \epsilon(-\mathbf{1} + S)\epsilon[\varphi' + \mathcal{M}_0\varphi]^* = \epsilon(-\mathbf{1} + S)\mathcal{F},$$

and thus

$$(\mathbf{1} - S) \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = (\mathbf{1} - S) \begin{pmatrix} \psi_c \\ \psi \end{pmatrix} = (\mathbf{1} - S) \begin{pmatrix} F_c \\ F \end{pmatrix} = 0, \quad (3.12)$$

which explicitly reflects the supersymmetry of the boundary conditions.

Concerning the action one can easily rewrite it as¹⁰

$$\begin{aligned} \mathcal{S} = & \int_{\Sigma} \left[i\bar{\psi}_c \bar{\sigma}^{\mu} \partial_{\mu} \psi_c + i\psi \sigma^{\mu} \partial_{\mu} \bar{\psi} - \phi_c^* \square \phi_c - \phi^* \square \phi + |F_c|^2 + |F|^2 \right. \\ & \left. + F_c(-\partial_5 + M)\phi + \phi_c(-\partial_5 + M)F + \psi_c(\partial_5 - M)\psi + \text{h.c.} \right] \\ & + \int_{\partial\Sigma} \left[K - \left(\frac{1}{2}s_3\psi_c\psi - \frac{1}{4}s_+\psi_c\psi_c + \frac{1}{4}s_-\psi\psi + \text{h.c.} \right) \right. \\ & \left. - \frac{1}{2}M\vec{p}_0 \cdot \vec{s}\varphi^{\dagger}(\mathbf{1} + S)\varphi - \frac{1}{2}(\varphi^{\dagger}S\varphi)' \right] \end{aligned} \quad (3.13)$$

where K comes from partial integration and is given by

$$K = \frac{1}{2}\partial_5(|\phi_c|^2 + |\phi|^2) + M(|\phi_c|^2 - |\phi|^2) - \frac{1}{2}(\psi_c\psi + \bar{\psi}_c\bar{\psi}) + \phi_c F + \phi_c^* F^*,$$

and in addition we have defined $s_{\pm} = s_1 \pm is_2$ and $\vec{s} = (s_1, s_2, s_3)$.

Notice that the bulk term of (3.13) is already $N = 1$ invariant without any boundary contribution, which implies that $\mathcal{S}'_{bd} = \mathcal{S}_{bd} + \int_{\partial\Sigma} K$ has to be so. Let us now explicitly check this point. The fermionic component of \mathcal{S}'_{bd} is given by

$$\int_{\partial\Sigma} \left[-\frac{1}{2}(1 + s_3)\psi_c\psi + \frac{1}{4}s_+\psi_c\psi_c - \frac{1}{4}s_-\psi\psi + \text{h.c.} \right] \quad (3.14)$$

while for the bosonic sector we find

$$\begin{aligned} & \int_{\partial\Sigma} \left[-\frac{1}{2}M\vec{p}_0 \cdot \vec{s}\varphi^{\dagger}(\mathbf{1} + S)\varphi + \frac{1}{2}(\varphi^{\dagger}(\mathbf{1} - S)\varphi)' + M\varphi^{\dagger}\sigma_3\varphi + \phi_c F + \phi_c^* F^* \right] \\ & = \int_{\partial\Sigma} \left\{ -\frac{1}{2}\varphi^{\dagger}(\mathbf{1} + S)[\varphi' + M\vec{p}_0 \cdot \vec{s}\varphi] + \frac{1}{2}\varphi^{\dagger}(\mathbf{1} - S)\varphi \right. \\ & \quad \left. + \varphi^{\dagger}\varphi' + M\varphi^{\dagger}\sigma_3\varphi + \phi_c F + \phi_c^* F^* \right\}. \end{aligned} \quad (3.15)$$

¹⁰For simplicity in the notation we omitted the subscript f .

Using now the boundary conditions and the equations of motion for the auxiliary fields

$$\int_{\partial\Sigma} \phi^* F_c^* + \phi_c F . \quad (3.16)$$

Then it can be easily checked that

$$\begin{aligned} \phi^* F_c^* + \phi_c F &= \frac{1}{2}(1 + s_3) (F_c \phi + \phi_c F) + \frac{1}{2} s_- \phi F - \frac{1}{2} s_+ \phi_c F_c + \text{h.c.} \\ &+ \frac{1}{2} \varphi^T \epsilon (1 + S) \mathcal{F} + \frac{1}{2} \mathcal{F}^\dagger \epsilon (1 + S^*) \varphi^* , \end{aligned} \quad (3.17)$$

where using the identities $\epsilon S \epsilon = S^* = S^T$ one immediately realizes that the last two terms separately vanish upon the use of boundary conditions. Thus, as claimed, we can write the whole action in terms of superfields as

$$\begin{aligned} S &= \int_{\Sigma} d^4\theta [\bar{W}W + \bar{W}_c W_c] - \int_{\Sigma} d^2\theta W_c (\partial_5 - M)W + \text{h.c.} \\ &+ \int_{\partial\Sigma} d^2\theta \left[\frac{1 + s_3}{2} W W_c + \frac{s_-}{4} W W - \frac{s_+}{4} W_c W_c \right] + \text{h.c.} \end{aligned} \quad (3.18)$$

A comment on gauge charges is in order here. The gauge charges assignment is not the usual one in the sense that if W transforms in the \mathfrak{R} representation, W_c lies in $\bar{\mathfrak{R}}$. Instead, one must find a representation of the gauge group where the reality constraints are preserved, as it is done in ref. [9].

Recalling that we have taken $\vec{p}_0 = (0, 0, 1)$, in order to have a general mass configuration we simply undo the $SU(2)_H$ rotation. Explicitly, (3.18) can be rewritten in a compact way as

$$\begin{aligned} S &= \int_{\Sigma} d^4\theta \bar{\mathcal{W}}\mathcal{W} - \frac{1}{2} \int_{\Sigma} d^2\theta [\mathcal{W}^T \epsilon \mathcal{W}' + M \mathcal{W}^T \epsilon \vec{p}_0 \cdot \vec{\sigma} \mathcal{W}] + \text{h.c.} \\ &- \frac{1}{4} \int_{\partial\Sigma} d^2\theta \mathcal{W}^T \epsilon S \mathcal{W} + \text{h.c.} \end{aligned} \quad (3.19)$$

where $\mathcal{W} = (W_c, W)^T$ (already $SU(2)_H$ covariant). Therefore an arbitrary $SU(2)_H$ rotation leaves the kinetic term invariant while the mass term is brought into the generic form

$$M \vec{p} \cdot \vec{\sigma} . \quad (3.20)$$

In fact this is not only the most general mass term compatible with the 4D $N = 2$ structure [15], but the most general one compatible with the 5D Lorentz invariance. In terms of a 4-component 5D Dirac spinor the most general mass term can be written as

$$\alpha \bar{\Psi} \Psi + \beta \Psi^T C \Psi + \beta^* \Psi^\dagger C \Psi^* , \quad (3.21)$$

with $\alpha \in \mathbb{R}$ and C the 5D charge conjugation matrix. One can easily check that (3.21) expressed in terms of 2-component Weyl spinors precisely yields a mass matrix of the form (3.20).

3.1 General boundary term

In this section we will briefly check that the boundary term previously displayed is indeed on-shell equivalent to the most general boundary term that can be written, which is

$$\tilde{\mathcal{S}}_{\text{bd}} = \int_{\partial\Sigma} d^2\theta \left[\frac{\mu}{2} W W + \frac{\lambda}{2} W_c W_c + \nu W W_c \right] + h.c. \quad (3.22)$$

where μ , λ and ν are arbitrary complex numbers. The variation of $\mathcal{S}_{\text{bk}} + \tilde{\mathcal{S}}_{\text{bd}}$ yields the boundary term

$$\int_{\partial\Sigma} d^2\theta [\delta W_c (\lambda W_c + \nu W) + \delta W (\mu W + \nu W_c - W_c)] + h.c.$$

which provides the boundary conditions

$$\mu W + \nu W_c - W_c = 0 \quad (3.23)$$

$$\lambda W_c + \nu W = 0. \quad (3.24)$$

One can easily check that in order to not overdetermine the system the complex parameters have to satisfy the relation

$$\mu \lambda - (\nu - 1)\nu = 0, \quad (3.25)$$

and that (3.23)–(3.24) are invariant under the redefinitions

$$W_c \leftrightarrow W, \lambda \leftrightarrow \mu, \nu \leftrightarrow 1 - \nu. \quad (3.26)$$

In the special case $\nu = 0$ the boundary conditions reduce to

$$\begin{cases} \lambda = 0, \mu W - W_c = 0 \\ \text{or} \\ \mu = 0, W_c = 0 \end{cases} \quad (3.27)$$

while the case $\nu = 1$ is obtained from the previous one by means of the redefinitions (3.26). In the general case $\nu \notin \{0, 1\}$, (3.23)–(3.24) reduce to

$$z W_c + W = 0$$

with $z = \lambda/\nu$. This means that we have a lot of redundancy in the parameters ν, μ, λ since only the complex number z plays a role in solving the boundary conditions. Actually by letting z to take any complex value we cover the whole set of boundary conditions including $\nu = 0$, which corresponds to $z \rightarrow \infty$. As a matter of fact, the parameterization

$$\nu_0 = \frac{1}{2}(1 + s_3), \mu_0 = \frac{1}{2}s_- = \frac{1}{2}\sqrt{1 - s_3^2} e^{i\delta}, \lambda_0 = -\mu_0^*,$$

verifies $\mu_0 \lambda_0 - (\nu_0 - 1)\nu_0 = \frac{1}{4}(1 - s^2) = 0$ and the mapping $z = \sqrt{\frac{1-s_3}{1+s_3}} e^{-i\delta}$ covers the whole complex plane.

4 Supersymmetry breaking by boundary terms

As we saw previously, supersymmetry is broken by the boundary terms whenever $\vec{t}_0 \neq \vec{t}_\pi$ and/or $N_f \neq \vec{p} \cdot \vec{s}_f M$. The misalignment of the R -matrices is equivalent to have a local $SU(2)_R$ transformation, $e^{iy\vec{\omega} \cdot \vec{\sigma}}$, such that $T_\pi = e^{i\pi\vec{\omega} \cdot \vec{\sigma}} T_0 e^{-i\pi\vec{\omega} \cdot \vec{\sigma}}$ which is a Scherk-Schwarz like breaking [9, 16–18] and therefore a (super) soft breaking. A very elegant proposal consists of breaking supersymmetry at the supergravity level via the expectation value acquired by some auxiliary field of the supergravity multiplet [19, 20].

We suggest a very similar breaking mechanism [13, 21] restricting to the case of flat space $\mathcal{M}^4 \times I$, where I is the interval $[0, \pi]$, with the metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - R^2 dy^2, \quad (4.1)$$

where R is the radion field which parametrizes the compact extra dimension labeled by y , which ranges from 0 to π . Supersymmetrization of the radion field is given by

$$T = R + iB_5 + \theta\Psi_R^5 + \theta^2 F_T, \quad (4.2)$$

where B_5 is the fifth component of the graviphoton, Ψ_R^5 is the fifth component of the right-handed gravitino and F_T is a complex auxiliary field. The supersymmetric action will be given by

$$S = \int_\Sigma d^4\theta \frac{T + \bar{T}}{2} \bar{\mathcal{W}}\mathcal{W} - \frac{1}{2} \int_\Sigma d^2\theta [\mathcal{W}^T \epsilon \mathcal{W}' + M T \mathcal{W}^T \epsilon \vec{p} \cdot \vec{\sigma} \mathcal{W}] + \text{h.c.} \\ - \frac{1}{4} \int_{\partial\Sigma} d^2\theta \mathcal{W}^T \epsilon S \mathcal{W} + \text{h.c.} \quad (4.3)$$

Supersymmetry can be spontaneously broken by allowing expectation values for the auxiliary field of the radion

$$\langle T \rangle = R + 2\omega \theta^2, \quad (4.4)$$

ω being a dimensionless constant. The bosonic sector of (4.3), disregarding the 4D kinetic term, reads¹¹

$$\int_\Sigma \mathcal{F}^\dagger \mathcal{F} - \left\{ -\omega \mathcal{F}^\dagger \varphi + \frac{1}{2} \mathcal{F}^T \epsilon \varphi' + \frac{1}{2} \varphi^T \epsilon \mathcal{F}' + M \mathcal{F}^T \epsilon \vec{p} \cdot \vec{\sigma} \varphi \right. \\ \left. + \frac{1}{2} M \omega \varphi^T \epsilon \vec{p} \cdot \vec{\sigma} \varphi + \text{h.c.} \right\} - \frac{1}{2} \int_{\partial\Sigma} \mathcal{F}^T \epsilon S \varphi + \text{h.c.} \quad (4.5)$$

Obviously, the boundary conditions are the same as before, that is

$$(\mathbf{1} - S) \varphi = (\mathbf{1} - S) \mathcal{F} = 0, \quad (4.6)$$

and since the new equations of motion for the auxiliary fields are

$$\mathcal{F} = \epsilon [\varphi' + M \vec{p} \cdot \vec{\sigma} \varphi]^* - \omega \varphi, \quad (4.7)$$

¹¹The fermions are unaffected by the radion VEV.

it is clear that (4.6) is equivalent to the system¹²

$$(\mathbf{1} - S) \varphi = 0, \tag{4.8}$$

$$(\mathbf{1} + S) [\varphi' + M \vec{s} \cdot \vec{p} \varphi] = 0, \tag{4.9}$$

or in the language of real formalism

$$(\mathbf{1} - \sigma_3 \otimes S) \Phi = 0, \tag{4.10}$$

$$(\mathbf{1} + \sigma_3 \otimes S) [\Phi' + M \vec{s} \cdot \vec{p} \Phi] = 0. \tag{4.11}$$

In addition, the bulk (bosonic) action can be written as

$$\int_{\Sigma} -\frac{1}{2} \Phi^\dagger \square \Phi + \frac{1}{4} \Phi^\dagger D_5^2 \Phi + \frac{1}{4} (D_5^2 \Phi)^\dagger \Phi + (\text{mass term}), \tag{4.12}$$

with $D_5 = \partial_5 + i\omega\sigma_2$ and thus by a local redefinition

$$\Phi \rightarrow \mathbf{1}_H \otimes (e^{-i\sigma_2 y})_R \Phi,$$

the connection is absorbed and the breaking takes place at the boundaries since now $T_\pi = e^{\pi i \sigma_2} T_0 e^{-\pi i \sigma_2}$.

In order to study the nature of the breaking due to the departure of N_f from $M \vec{s}_f \cdot \vec{p}$ we shall consider the boundary action (3.18) plus an effective coupling such that the new boundary term is given by

$$\begin{aligned} & -\frac{1}{4} \int_{\partial\Sigma} d^2\theta \mathcal{W}^T \epsilon S \mathcal{W} + \text{h.c.} \\ & -\frac{1}{\Lambda^3} \int_{\partial\Sigma} d^4\theta \frac{1}{2} (\bar{\mathcal{N}}'_f \mathcal{N}_f + \bar{\mathcal{N}}_f \mathcal{N}'_f) \bar{\mathcal{W}} \mathcal{W}, \end{aligned} \tag{4.13}$$

with Λ the scale of the cutoff and $\mathcal{N}_f, \mathcal{N}'_f$ localized superfields whose auxiliary fields acquire VEVs, say F_f, F'_f , such that $\frac{\bar{F}'_f F_f}{\Lambda^3} = N_f$. Now the (bosonic) boundary conditions¹³ turn into

$$(\mathbf{1} - S_f) \varphi = 0, \tag{4.14}$$

$$(\mathbf{1} - S_f) \mathcal{F} - 2N_f \epsilon \varphi^* = 0, \tag{4.15}$$

with \mathcal{F} given by (3.5). Finally, using the identity $\epsilon S \epsilon = S^*$ we are left with

$$(\mathbf{1} - S_f) \varphi = 0, \tag{4.16}$$

$$(\mathbf{1} + S_f) [\partial_5 + \vec{s}_f \cdot \vec{p} M + N_f] \varphi = 0, \tag{4.17}$$

This shows explicitly that this breaking has a soft nature, a result which the calculation of the radiative corrections coming from such breaking term to the Higgs mass coupling strongly suggests [9].

¹²The boundary conditions are the same because we are working in a Hosotani like basis.

¹³The fermionic boundary conditions are unaffected.

5 Conclusions

The main objective of the present work was to explicitly show the translation into superfield language of a model for ElectroWeak Symmetry Breaking (EWSB) developed in component fields subject to reality constraints. Albeit the model was proven to be (on-shell) supersymmetric under certain bulk-brane configurations, that being broken by boundary terms, it was not totally evident how to develop an equivalent off-shell supersymmetric model. To build such a dictionary was motivated by several reasons: The model predicted a very interesting scenario where a tachyon mode for the Higgs was present at tree level. This opened a chance for the EWSB to be triggered by the negative top-stop corrections, since the negative squared mass could partially cancel the positive gauge corrections. For that, however, an exhaustive study of the quantum behavior of the model is needed but, unfortunately, to embed interacting terms within the real formalism is not an easy task. On the other hand, the breaking of supersymmetry comes from the misalignment between several bulk-brane parameters, one of them being easily identified with a Scherk-Schwarz like breaking, and coming both from boundary terms it indicates a spontaneous mechanism, nevertheless, a explicit translation into superfield formalism helps to clarify its nature.

It is worth remarking, however, that the dictionary we have developed so far does not mean to be neither a formal proof nor a consistent extension of the model, at any level. For that, among other aspects, one should justify the presence of the spurion fields breaking the supersymmetry through their vacuum expectation values.

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A Supersymmetry of boundary conditions

The (on-shell) variations of the boundary conditions, eqs. (2.10), (2.11), (2.12), are given by

$$\begin{aligned}
 (\mathbf{1} + R) \delta_\chi \Phi &= -\bar{\chi} \gamma^{\dot{5}} (i\gamma^{\dot{5}} - S) \Psi + i (\bar{\chi} i\gamma^{\dot{5}} + T \bar{\chi}) S \Psi, \\
 (-\mathbf{1} + R) [\delta_\chi \Phi' + N \delta_\chi \Phi] &= i (\bar{\chi} i\gamma^{\dot{5}} + T \bar{\chi}) \mathcal{M} \Psi + T \bar{\chi} \gamma^{\dot{5}} \mathcal{M} (i\gamma^{\dot{5}} - S) \Psi \\
 &\quad - (\bar{\chi} i\gamma^{\dot{5}} + T \bar{\chi}) \not{\partial} \Psi - iT \bar{\chi} \gamma^{\dot{5}} \not{\partial} (i\gamma^{\dot{5}} - S) \Psi \\
 &\quad + i(N - M \vec{p} \cdot \vec{s}) T \bar{\chi} (i\gamma^{\dot{5}} + S) \Psi \\
 &\quad - iN (\bar{\chi} i\gamma^{\dot{5}} + T \bar{\chi}) i\gamma^{\dot{5}} \Psi,
 \end{aligned}$$

$$\begin{aligned}
(-i\gamma^{\dot{5}} + S) \delta_\chi \Psi &= -\gamma^\mu (i\gamma^{\dot{5}}\chi - \chi T) \partial_\mu \Phi - \not{\partial} \chi T (\mathbf{1} + R) \Phi \\
&+ (i\gamma^{\dot{5}}\chi - \chi T) \mathcal{M} \Phi + \chi T \mathcal{M} (\mathbf{1} + R) \Phi \\
&+ i\gamma^{\dot{5}}\chi T (-\mathbf{1} + R) [\Phi' + M\vec{p} \cdot \vec{s} \Phi] \\
&+ \gamma^{\dot{5}} (i\gamma^{\dot{5}}\chi - \chi T) \Phi',
\end{aligned}$$

where we have used the (bulk) equations of motion for the fermions as well as those for the auxiliary fields. Furthermore, we have omitted the R and H indices as well as the subscript f . The above variations cancel upon the restrictions

$$N_f = M\vec{p} \cdot \vec{s}_f, \quad (i\gamma^{\dot{5}} - T^T) \chi = 0. \quad (\text{A.1})$$

B $N = 1$ splitting

To complete the dictionary between the real and the superfield descriptions we will briefly give the splitting of the 5D hypermultiplet into 4D superfield pieces. Being $T = -\sigma_3$ the projection on the supersymmetry parameters, eq. (A.1), reads

$$(\mathbf{1} + \sigma_3 \otimes i\gamma^{\dot{5}}) \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} (\mathbf{1} + \gamma^{\dot{5}}) \chi_1 \\ (\mathbf{1} - \gamma^{\dot{5}}) \chi_2 \end{pmatrix} = 0, \quad (\text{B.1})$$

with

$$\chi_1 = \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix},$$

as a consequence $\xi = 0$ and the supersymmetric transformations, eq. (2.14), can be written as

$$\begin{aligned}
\delta_\eta (-i\Phi_2^1)^* &= \eta (-\psi_R^1) \\
\delta_\eta (-\psi_R^1) &= -i\sigma^\mu \bar{\eta} \partial_\mu (-i\Phi_2^1)^* + \eta (-2F_1^1 + i\partial_5 \Phi_1^1)^* \\
\delta_\eta (-2F_1^1 + i\partial_5 \Phi_1^1)^* &= -i\bar{\eta} \bar{\sigma}^\mu \partial_\mu (-\psi_R^1)
\end{aligned} \quad (\text{B.2})$$

$$\begin{aligned}
\delta_\eta (-i\Phi_1^1) &= \eta (\psi_L^1) \\
\delta_\eta (\psi_L^1) &= -i\sigma^\mu \bar{\eta} \partial_\mu (-i\Phi_1^1) + \eta (-2F_2^1 - i\partial_5 \Phi_2^1) \\
\delta_\eta (-2F_2^1 - i\partial_5 \Phi_2^1) &= -i\bar{\eta} \bar{\sigma}^\mu \partial_\mu (-i\psi_L^1)
\end{aligned} \quad (\text{B.3})$$

which corresponds to the pair of chiral superfields [6]

$$W = i\Phi_2^{1*} + \sqrt{2}\theta (-\psi_R^1) + (-2F_1^{1*} - i\partial_5 \Phi_1^{1*}) \theta^2, \quad (\text{B.4})$$

$$W_c = (-i\Phi_1^1) + \sqrt{2}\theta \psi_L^1 + (-2F_2^1 - i\partial_5 \Phi_2^1) \theta^2. \quad (\text{B.5})$$

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